Preservice teachers' use of spatio-visual elements and their level of justification dealing with a geometrical construction problem

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Abstract: The main purpose of this research is to determine to what extent preservice teachers use visual elements and mathematical properties when they are dealing with a geometrical construction activity. The axiomatic structure of the Euclidian geometry forms a coherent field of objects and relations of a theoretical nature; and thus it constitutes a favorable ground for the learning of the reasoning and justification. In geometry, even if the reasoning is assisted by concrete elements such as drawings, it is not based on actions or spatio-visual elements. Arguments based on mathematical properties are required in geometrical reasoning. A successful teaching of the geometry directly depends on the subject knowledge of the teachers. Another purpose is to map out the different kinds of justifications implemented by teachers related to such an activity. Analyses of the responses indicate that a large proportion of preservice teachers use visual elements for their geometrical constructions and they are tended to use naive empiricism concerning their justifications.

Key words: geometrical constructions; justification; spatial and geometrical properties; preservice teachers

1. Introduction

There exists a gap between the role of geometry in mathematics, in its history and the place of geometry in Turkish National Elementary School Curriculum. In fact, although geometry plays an important role in mathematics teaching and in history of mathematics, it takes a restricted part of the Turkish National Elementary School Curriculum. Despite this limited place, mathematicians as well as mathematics educators agree that geometry should be an important part of education (Leher & Chazan, 1998).

In fact, geometry forms a natural field for the learning of the reasoning, rigueur, exactness, justification and proof. These notions are not only very important for a scientific formation, but they are also omnipresent in the human interactions within the society. Thus they seem to be also important concerning a citizen formation (Kahane, 2002).

For not to be misunderstood, it should be underlined that reducing the reasoning to the domain of the geometry is evidently not sufficient for a rational argumentation formation as indicated by the college group of IREM d'Aix-Marseille (GRT, 2000). But, the axiomatic structure of the Euclidian geometry constitutes a favorable ground for the learning of the reasoning and argumentation.

The successful teaching of geometry at the elementary school depends crucially on the subject knowledge of teachers. Many studies reveal the difficulties teachers face if they are uncertain or if the content unfamiliar to them (Ball & Bass, 2003).

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In geometry, reasoning is not only based on words or on symbols, but also on drawings and visual images (mental pictures). In addition to this, the reasoning controls not only the relationship of these visual images to the ideal geometrical objects but also the constructions or handling of drawing tools (Richard, 2000).

In deed, diagrams in two dimensional geometry play an ambiguous role: on the one hand they refer to theoretical geometrical properties, while on the other, they offer spatio-graphical properties that can give rise to a students perceptual activity (Laborde, 2005).

Researches indicate that the most encountered difficulty in the learning of the geometrical reasoning is to leave off the concrete drawing. Hence, it becomes important to throw light on the concrete drawing's role for the teachers and visual elements' usage while realizing a geometrical construction. It seems also important to find out how teachers are situated in a typology related to reasoning and proof in geometry. In this study it has been tried to find out how student teachers are situated among proof levels indicated by Balacheff (1988).

The research questions of this study are to be formulated as fallows:

(1) Q1: What is the role, for the primary school preservice teachers, of concrete drawing and visual elements' usage while realizing a geometrical construction?

(2) Q2: How student teachers are situated among proof levels indicated by Balacheff (1988)?

2. Methodology

The sample of this research consists of 60 preservice elementary teachers at Uludag University, Faculty of Education in Bursa, Turkey. The participants were third grade students who were taking mathematics teaching course. In the experimental part of the study, all of the participants responded the geometrical construction problem indicated below:

Draw an equilateral triangle. Explain your drawing. Demonstrate that your triangle is an equilateral triangle.

Student teachers were only allowed to use compass and straightedge for their construction. No time limitation was imposed. This construction problem is at the elementary school level and the student teachers have to teach the construction once nominated as teachers.

3. Analyses of data

The analyses of the experimental part are planned in two perspectives. One of them took into account student teachers' drawings and the other one was related with their written justifications.

In the analyses, the justifications of students are characterized by using Balacheff's hierarchical proof levels: naïve empiricism, crucial example, generic example, and thought experiment (Balacheff, 1987). In the case of naïve empiricism and crucial example, considered as pragmatic proofs, statements are validated by concrete or mental actions. Naïve Empiricism concerns an inductive perspective where conclusions are based on small number of cases. For the Crucial Experiment, the question of generalization is considered with the examination of extreme cases. Concerning generic example and thought experiment, considered as intellectual proofs, arguments are based on concepts and language. Intellectual proofs are not necessarily formal, but they are detached from concrete actions (Knipping, 2001). For the Generic example, the proposition is proved by examining a prototypical case and appealing to the structural properties of mathematics. Thought experiment differs from the proof by generic example in that instead of a prototypical case an abstract general case is examined (see Table 1).

Table 1 Hierarchical proof levels of Balacheff

Proof levels	
Pragmatic proofs	Naïve empiricism
	Crucial example
Intellectual proofs	Generic example
	Thought experiment

This categorization of different types of proofs given by Balacheff contributed to our research for the analyses of the preservice teachers' answers concerning the question related to the justification of their construction.

Student teachers' drawings were coded into two main categories concerning the analyses of their responses. These categories were mathematically acceptable and mathematically not acceptable. And besides these two categories another category was distinguished in the case of no response or non coherent explanation. Based on a priori analyses of possible response-types, the main categories were divided into several subcategories. Table 2 shows this categorization of student teachers' drawing responses (see Table 2).

Table 2 Categorizations of drawing responses

Mathematically acceptable	Mathematically not acceptable	Failed
Euclidean method	Proper mathematical property but visual construction	
Radius length chord method	Non proper mathematical property	
Regular hexagon method	Purely visual construction	
Circumferential circle method		

The subcategories of mathematically acceptable responses were "Euclidean method", "radius length chord method", "regular hexagon method" and "circumferential circle method".

The category called "Euclidean method" is concerned constructions based on the one that Euclid provides in The Elements as the first proposition of the first book. In this construction method; two circles are drawn with the same radius with a point from each circle intersecting the radius of the other circle. And then, lines are drawn from the center points to the intersection point of the two circles and between the two centers (see Figure 1).

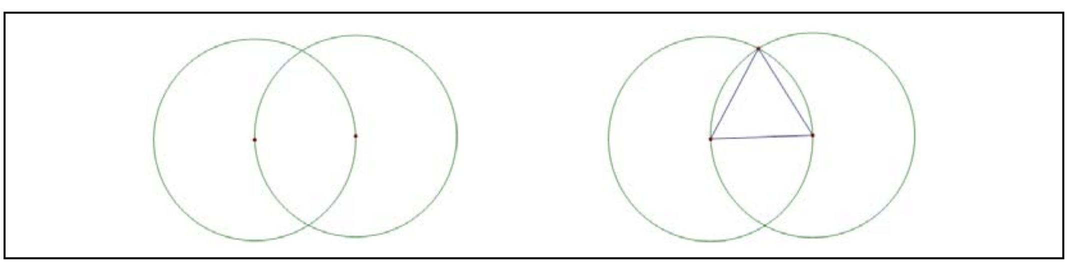


Figure 1 Construction with Euclidean method

Constructions classified as "radius length chord method" are those obtained by constructing a chord in a circle with the same length of the radius and then joining the center of the circle with the end points of this chord (see Figure 2).

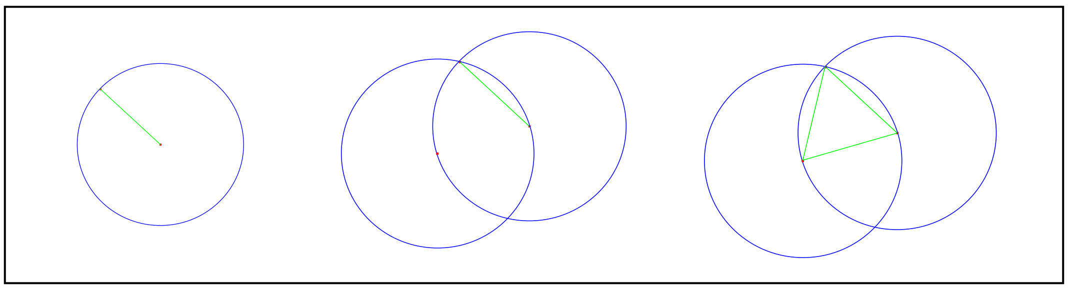


Figure 2 Construction with radius length chord method

The construction method called "regular hexagon method" is the one which is made by constructing a circle, then marking the radius off six times around the circumference. The vertexes of the triangle are then the three non-adjacent marks of the compass on the circumference.

This last construction method is taught at the elementary school level. That is, elementary school teachers have to teach this method to their students (see Figure 3).

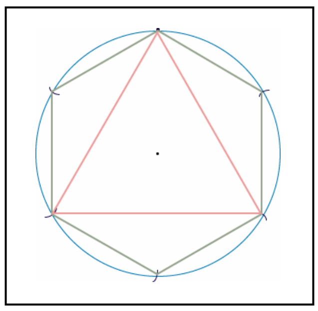


Figure 3 Construction with regular hexagon method

And, the "circumferential circle method" construction method consists of drawing a diameter of a circle OL and then constructing its perpendicular bisector AK. Bisect OK in point M, and extend the line BC through M. The resulting figure ABC is then an equilateral triangle (see Figure 4).

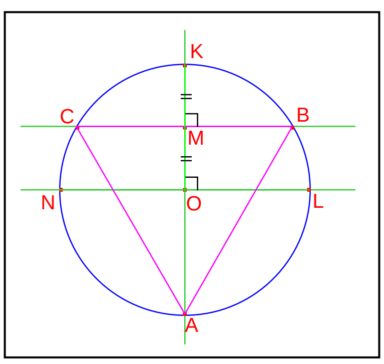


Figure 4 Construction with circumferential circle method

The subcategories of mathematically not acceptable responses were "Proper mathematical property but visual construction", "Non proper mathematical property" and "Purely visual construction".

Constructions classified as "Proper mathematical property but visual construction" are those constructed using spatial properties or visual elements but related explanations are given by mathematical properties.

The category called "Non proper mathematical property" concerns non correct construction where the explanation is given by a geometrical property not appropriate to the construction problem.

Constructions classified as "Purely visual construction" are those based on perception (with an explanation based on perception).

4. Results

When categorizing students' drawings there were times where it was not possible to assign a drawing into a category without taking into account the related written explanation. Table 3 which was prepared in these conditions, shows the frequency of drawing responses with respect to the a-priori categorizations resulting from the analyses (see Table 3).

Table 3 Frequency of drawing responses with respect to the a-priori categorizations

Mathematically acceptable	(f)	Mathematically not acceptable	(f)	Failed
Euclidean method	7	Proper mathematical property but visual construction	17	4
Radius length chord method	2	Non proper mathematical property	4	
Regular hexagon method	-	Purely visual construction	26	
Circumferential circle method	-			

Analyses of teacher students' drawings and explanations bring to light the important usage of the perception for geometrical constructions (26 constructions on 60 classified as purely visual). The following paper in Figure 5 shows one of the most striking examples of perception's usage (see Figure 5).

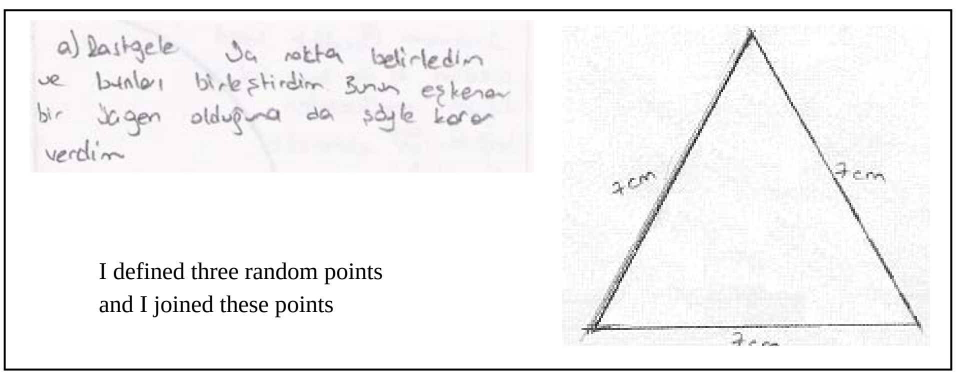


Figure 5 Student explanation for how he/she drew an equilateral triangle

Student, in his explanation for how he drew the triangle, write that he has chosen the three points randomly. Although the student express that he has drawn the triangle by choosing the three points randomly, in fact his drawing is purely based on perception.

We also notice the dominance of the perception in the constructions by the raised number of students' answers is classified in the "proper mathematical property but visual construction" category as seen in Table 3. Indeed, mathematically acceptable constructions appear only at few students' responses. The construction method "regular hexagon method", at the level of the elementary school that they will have to teach, and the construction method "Circumferential circle method" do not appear.

Analyses of the preservice teachers' answers concerning the question related to the justification of their

construction which were classified according to Balacheff's proof levels are shown in Table 4.

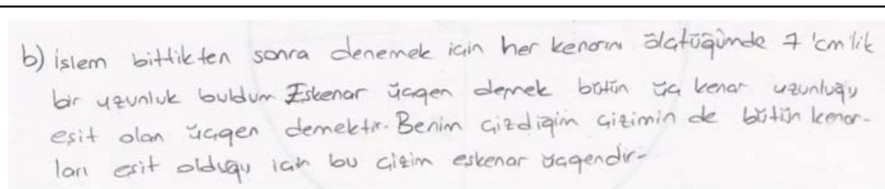
Table 4 Frequency of justification responses

Proof levels	(f)
Naïve empiricism	25
Crucial example	6
Generic example	6
Thought experiment	1
Failed	6
No answer	16

Although Balacheff's proof levels were categorized at four levels, through the analysis of students justifications the necessity of adding the categories "failed" and "no answer" occurred. "Failed" indicates that student's related explanation is not coherent with the construction problem. The most noticeable result of these analyses which can be also seen from the Table 4 is that student teachers' responses accumulated on naive empiricism. These responses rarely took the form of crucial example and generic example.

The corresponding analysis using frequencies shows that preservice teachers judge a justification sufficient when used on a unique example. Most of the justifications are made through careful measurements on the drawing and the student teachers do not express an argument for the generalization of their justification.

In the example shown in Figure 6, the student consider measurement adequate as a justification of his drawing.

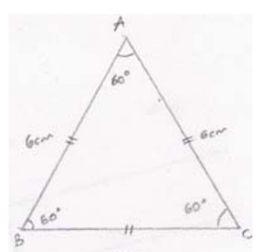


After completing (the construction), when I measured all sides to try, I found a length of 7 cm. Equilateral triangle means a triangle which is three sides' length are equal. As the triangle that I have drawn has all sides equal, this drawing is an equilateral triangle.

Figure 6 A student's justification response

And another example where the idea of measurement is conceived as the base for justification is given in the Figure 7:

We can prove that our triangle is an equilateral triangle by measuring its interior angles.
We can support this, by the equality of its side lengths.



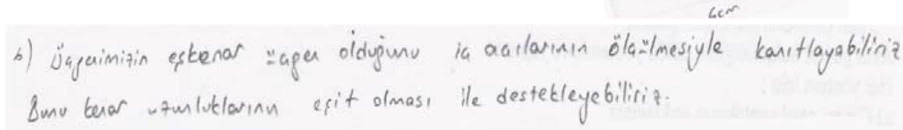


Figure 7 A student's justification response

During this research, the same behaviors as Laborde (2005) underlines have also been remarked on primary

preservice teachers.

"When students are asked by a teacher to construct a diagram, the teacher expects them to work at the level of geometry using theoretical knowledge, whereas students very often stay at a graphical level and try only to satisfy the visual constraints" (Laborde, 2005).

In deed, for drawing responses as well as for justification responses, a majority of the preservice teachers worked on the concrete drawing and not really on the geometrical properties and relations represented by the drawing.

5. Conclusions

One dimension of our research was to determine whether preservice teachers were able to make a geometrical construction at the elementary school level that they will have to teach as teachers. Other dimension was look for how the student teachers were situated concerning justifications connected to geometrical construction activities.

In spite of the fact that the student teachers had sufficient mathematical knowledge, analyses of their responses showed a restricted number of construction using geometrical properties (and not those spatio-visuals), Indeed, it can be thought that preservice teacher have restricted achievements on transmitting their geometrical knowledge to an activity of construction or a task of justification.

This research showed that preservice teachers use excessively visual elements (for realizing geometrical constructions as well as for formulating justifications). It can be supposed that this difficulty may result from the fact that an interrelation between usage of the geometrical tools and the formal mathematical knowledge was not established for the student teachers. Additionally, preservice teachers maybe could not manage to organize their reasoning in order to succeed in the geometrical construction using the appropriate mathematical properties.

All these results show a deficiency concerning construction tasks and geometrical reasoning; and brings to light the importance of researches, in the future, on this domain to analyze more exactly causes of this deficiency noticed on the preservice teachers and to find solutions for filling this gap.

Finally, we can say from our research that geometry should lie more in the design of the National Curriculum and at university level; this can only be realized through researches showing the importance, on the reasoning, of the geometry learning and geometrical justifications.

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